

Supplemental Material for “Precision Hypernuclear Structure from Interaction Cross Sections”

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I. THREE-BODY WAVE FUNCTIONS

The hypertriton is treated as $p+n+\Lambda$ in Jacobi coordinates $\mathbf{x} = \mathbf{r}_p - \mathbf{r}_n$ and $\mathbf{y} = \mathbf{r}_\Lambda - (\mathbf{r}_p + \mathbf{r}_n)/2$. The variational wave function is expanded in correlated Gaussians over a broad logarithmic range of length scales. The basis includes the dominant deuteron-like pn S wave, a deuteron D -wave component, and correlated x - y terms. The YN strength is varied to span $0.10 \leq B_\Lambda \leq 0.50$ MeV while preserving the short-distance structure. Table I gives the validation point used in the main text.

TABLE I. Representative hypertriton wave-function properties for $B_\Lambda = 0.133$ MeV. Jacobi-coordinate radii are distinct from one-body center-of-mass radii.

Quantity	Value
E_{total}	-2.357 MeV
E_d^{model}	-2.224 MeV
B_Λ^{model}	0.1329 MeV
B_Λ^{tail}	0.1300 MeV
One-body proton rms radius	4.1328 fm
One-body neutron rms radius	4.132 fm
One-body hyperon rms radius	6.973 fm
One-body matter rms radius	5.253 fm
$r_{\Lambda-NN}$	10.34 fm
r_{NN}	2.960 fm
$r_{N-\Lambda N}$	3.917 fm
r_{geo}	4.498 fm

The radii increase from about 2.8 fm at $B_\Lambda = 0.50$ MeV to 5.7 fm at 0.10 MeV. Their ratio, $r_m(0.10)/r_m(0.50) \simeq 2.1$, is close to the universal weak-binding estimate $\sqrt{0.50/0.10} = 2.24$.

II. FINITE-RANGE OPTICAL-LIMIT GLAUBER MODEL

The projectile one-body densities are folded with the ^{12}C density and finite-range elementary profile functions. Schematically,

$$\Omega_{ab}(\mathbf{b}, \xi) = \sum_{j \in P} \int d^2s T_T(\mathbf{s}) \Gamma_{ab}^{(jT)}(\mathbf{b} + \mathbf{s}_j - \mathbf{s}), \quad (1)$$

where $a, b \in \{\Lambda, \Sigma\}$ and the nucleon contributions are diagonal in hyperon space. The elementary profiles are parameterized by total cross sections, real-to-imaginary ratios, and finite-range slopes. The central calculation uses high-energy NN , ΛN , and ΣN systematics at 1.5 GeV/nucleon; the $\Lambda N \leftrightarrow \Sigma N$ transition strength is varied independently. The Monte Carlo bands sample the elementary cross sections, profile slopes, real parts, target density, and finite-range parameters. This is the optical-limit approximation referred to in the main text.

The coupled-channel operator is

$$\hat{S} = e^{-\hat{\Omega}/2}, \quad \hat{\Omega} = \begin{pmatrix} \Omega_{\Lambda\Lambda} & \Omega_{\Lambda\Sigma} \\ \Omega_{\Sigma\Lambda} & \Omega_{\Sigma\Sigma} \end{pmatrix}. \quad (2)$$

For a real symmetric 2×2 matrix, defining

$$\bar{\Omega} = \frac{\Omega_{\Lambda\Lambda} + \Omega_{\Sigma\Sigma}}{2}, \quad \Delta = \frac{\Omega_{\Lambda\Lambda} - \Omega_{\Sigma\Sigma}}{2}, \quad q = \sqrt{\Delta^2 + \Omega_{\Lambda\Sigma}^2}, \quad (3)$$

one obtains

$$S_{\Lambda\Lambda} = e^{-\bar{\Omega}/2} \left[\cosh \frac{q}{2} - \frac{\Delta}{q} \sinh \frac{q}{2} \right], \quad (4)$$

$$S_{\Sigma\Lambda} = -e^{-\bar{\Omega}/2} \frac{\Omega_{\Lambda\Sigma}}{q} \sinh \frac{q}{2}. \quad (5)$$

Hence the survival and conversion probabilities depend on all opacity-matrix elements, not only on $\Omega_{\Lambda\Lambda}$.

III. CHANNEL DECOMPOSITION

For the incoming state $|\Psi_0, \Lambda\rangle$,

$$P_{\text{el}}(b) = |\langle \Psi_0, \Lambda | \hat{S} | \Psi_0, \Lambda \rangle|^2, \quad \sigma_I = \int d^2b [1 - P_{\text{el}}(b)]. \quad (6)$$

The Good-Walker diffractive breakup term is

$$\sigma_{\text{breakup}} = \int d^2b [\langle |S_{\Lambda\Lambda}|^2 \rangle_0 - |\langle S_{\Lambda\Lambda} \rangle_0|^2], \quad (7)$$

and explicit conversion is

$$\sigma_{\Lambda\Sigma} = \int d^2b \langle |S_{\Sigma\Lambda}|^2 \rangle_0. \quad (8)$$

The residual loss defines

$$\sigma_{\text{abs}} = \sigma_I - \sigma_{\text{breakup}} - \sigma_{\Lambda\Sigma}, \quad (9)$$

so the three terms are exclusive by construction and satisfy the exact sum rule used in the main text.

TABLE II. Channel decomposition. All cross sections are in mb.

B_Λ (MeV)	σ_{breakup}	$\sigma_{\Lambda\Sigma}$	σ_{abs}	σ_I
0.10	201	140	806	1147^{+47}_{-52}
0.15	148	132	748	1028^{+44}_{-44}
0.20	134	128	698	950^{+44}_{-44}
0.25	114	124	667	895^{+41}_{-39}
0.30	98.3	121	629	858^{+34}_{-34}
0.35	92.2	119	619	830^{+32}_{-33}
0.40	82.2	117	601	802^{+30}_{-27}
0.45	69.9	116	589	775^{+28}_{-26}
0.50	59.2	114	576	750^{+22}_{-28}

TABLE III. Calibration values used for $\sigma_I(r_m)$.

r_m (fm)	5.71	4.71	4.13	3.73	3.44	3.22	3.04	2.89	2.77
B_Λ (MeV)	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
σ_I (mb)	1147	1028	950	895	858	830	802	775	750

IV. UNCERTAINTY PROPAGATION AND COVARIANCE

The total uncertainty is propagated by Monte Carlo sampling of NN , ΛN , ΣN , $\Lambda N \leftrightarrow \Sigma N$, density, and finite-range/profile inputs. The covariance share of channel i is defined by $100 \text{Cov}(\sigma_i, \sigma_I) / \text{Var}(\sigma_I)$. This quantity can be negative because channel fluctuations are correlated and the channel sum is constrained.

The absorption channel accounts for approximately 80–90% of the total variance, the $\Lambda\Sigma$ channel for roughly 10–20%, and breakup has a small net covariance share. These percentages refer to uncertainty propagation, not to the central channel strengths.

V. BAYESIAN PRIOR AND BOUNDARY EFFECT

The posterior is $p(B_\Lambda|D) \propto p(D|B_\Lambda)p(B_\Lambda)$, with a uniform prior on $0.1 \leq B_\Lambda \leq 0.5$ MeV. The likelihood combines the experimental error and the interpolated theory covariance. Near the endpoints, the prior truncates one side of the likelihood, reducing the inferred credible width. Repeating the inference with a broader interval would raise the endpoint widths and flatten the broad maximum, but it would not remove the large information content set locally by $\delta B_\Lambda \sim \delta\sigma_I / |d\sigma_I/dB_\Lambda|$.

TABLE IV. Monte Carlo uncertainty budget. Widths are in mb.

B_Λ (MeV)	$\Delta\sigma_I$ (mb)	$\Delta\sigma_{\text{breakup}}$ (mb)	share (%)	$\Delta\sigma_{\Lambda\Sigma}$ (mb)	share (%)	$\Delta\sigma_{\text{abs}}$ (mb)	share (%)
0.10	50.1	56.0	-1.8	37.7	11.9	74.7	89.9
0.15	47.3	43.6	-0.7	35.0	15.9	63.7	84.9
0.20	43.1	32.7	0.1	33.6	13.2	57.0	86.7
0.25	38.4	28.7	-0.3	32.1	19.3	50.8	80.9
0.30	33.2	26.1	0.5	32.6	12.9	48.7	86.6
0.35	32.0	20.5	2.0	31.5	15.0	43.3	83.0
0.40	28.2	19.7	-1.4	30.6	20.3	42.6	81.1
0.45	28.0	16.2	-2.0	30.0	14.6	40.8	87.4
0.50	25.5	14.8	-5.0	29.8	15.3	40.4	89.7

TABLE V. Selected covariance matrices, in mb^2 .

$B_\Lambda = 0.10$ MeV	breakup	$\Lambda\Sigma$	absorption
breakup	3140.3	-490.7	-2695.2
$\Lambda\Sigma$	-490.7	1418.6	-627.9
absorption	-2695.2	-627.9	5581.4
$B_\Lambda = 0.30$ MeV	breakup	$\Lambda\Sigma$	absorption
breakup	680.9	-89.4	-586.3
$\Lambda\Sigma$	-89.4	1060.1	-828.3
absorption	-586.3	-828.3	2371.2
$B_\Lambda = 0.50$ MeV	breakup	$\Lambda\Sigma$	absorption
breakup	218.3	6.5	-257.3
$\Lambda\Sigma$	6.5	888.3	-794.8
absorption	-257.3	-794.8	1636.1

VI. COULOMB BREAKUP

The long-range Coulomb eikonal phase cannot be inserted naively because it produces the familiar divergence in first-order breakup amplitudes. For the light ^{12}C target used here, a separate finite Coulomb-breakup calculation gives 8.57 mb at 1.5 GeV/nucleon, much smaller than both the nuclear interaction cross section and its propagated uncertainty. Coulomb breakup is therefore omitted from the quoted σ_I . The corresponding value for ^{208}Pb is about 1.39 b; heavy targets require an explicit finite Coulomb treatment and lie outside the present scope.

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